IX.

WAVE LENGTHS OF ELECTRICITY ON IRON WIRES.

By Charles E. St. John, A. M.

Presented by Professor John Trowbridge, May 9, 1894.

Since the experimental demonstration of the existence of the oscillating electric discharge, it has been an interesting field of investigation to ascertain whether the magnetization of iron and nickel can follow such rapidly alternating impulses as are obtained by the oscillating discharge of a condenser through a circuit of low self-induction, and, if magnetization does follow, in what way and to what extent can it affect the character of an electric wave propagated along wires of magnetic material.

The results of investigation have shown considerable disagreement, as will be seen from the following brief *résumé* of the investigations bearing upon these points. The questions referred to did not distinctly appear in all the investigations, as they have arisen since; but results were obtained and published which directly relate to at least one of the points under consideration.

M. Savary announced, as early as 1826, that, when a needle was placed in a spiral through which a Leyden jar was discharged, reversals of polarity were obtained by varying the quantity of discharge through the spiral; and Faraday * adduces the magnetizing of needles and bars by common (static) electricity as evidence of its identity with Voltaic electricity, and in his experiment to show that common electricity can deflect the magnetic needle, when a Leyden battery is discharged through the galvanometer, he states the fact that the magnetism of the needle may be removed or reversed by the discharge.

Professor Henry † repeated the experiments of Savary with great skill and care. He obtained reversals of polarity by increasing the quantity of electricity discharged through the spiral in which the needle was placed, while the direction of the discharge remained the same, and by varying the distance between the primary and secondary.

* Experimental Researches on Electricity. 1833.

† Writings of Joseph Henry, p. 201. 1842.

This anomalous result was referred by Professor Henry to an action of the discharge of a Leyden jar never before recognized. He here first describes the oscillating character of such a discharge as follows: —

"The discharge, whatever may be its nature, is not correctly represented (employing for simplicity the theory of Franklin) by the single transfer of an imponderable fluid from one side of the jar to the other; the phenomena require us to admit the existence of a principal discharge in one direction, and then several reflex actions backward and forward, each more feeble than the preceding, until equilibrium is obtained. All the facts are shown to be in accordance with this hypothesis, and a ready explanation is afforded by it of a number of phenomena which are found in the older works on electricity, but which have until this time remained unexplained."

The apparent change in the direction of the induced currents with a change in distance between the primary and secondary circuit, as indicated by a change in the direction of the magnetization of the needle, was shown to be due to the fact that the discharge of the Leyden jar does not produce an induced current in a single direction, but several successive currents in opposite directions.

There can be no doubt that these discharges were oscillatory in character, and that steel needles and bars were magnetized by them, sometimes by the direct discharge, sometimes by the current induced in a neighboring circuit, sometimes by the first impulse, sometimes by the second or return impulse.

Feddersen * was of the opinion that iron might show some deviation from the behavior of copper and lead; of the last two he says, that the time between two consecutive like-directed current maxima is independent of the cross section and the specific conducting power of the wires forming the circuit, and also of the density of the accumulated electricity. And in regard to iron he adds the following note: "Beim Eisen könnte in Folge der Magnetisirungen eine Abweichung hervortreten: indess zeigt der Versuch dass dieselbe keinenfalls bedeutend ist, übrigens in dem Sinne erfolgen müsste, als wenn die Electricität beim Eisen ein grösseres Hinderniss fände als bei den übrigen Metallen."

The rate of oscillation obtained by Feddersen was one million per second.

The late Professor Hertz † gives in his first paper some experiments that bear upon this subject. He was of the opinion that an

* Poggendorff, Annalen, CVIII. 499. 1859. † Ibid., XXXI. 429. 1887.

iron wire in an oscillating circuit might be equivalent to a copper wire of greater length, owing to the higher self-induction of iron. He based his opinion upon the known fact that for slowly oscillating currents the self-induction of iron is eight or ten times greater than that of a copper wire of the same dimensions.

He says: "I therefore expected that short iron wires would produce equilibrium with longer copper wires. This expectation was not confirmed; the branches remained in equilibrium when the copper wire was replaced by an iron wire of equal length. If the theory of the observations here given is correct, this can only mean that the magnetism of iron is quite unable to follow oscillations so rapid as those with which we are here concerned, and that it therefore is without effect." *

The rate of oscillation here used was approximately one hundred million per second, and the diameter of the wires was two millimeters.

In the same paper, he gives another experiment of like tenor. He brought the primary and secondary into resonance, and then in one instance he surrounded one side of the rectangular secondary by an iron tube, and in a second instance he replaced this side by an iron wire of the same diameter as the copper wire. In each case he found the secondary still in resonance with the primary, and was confirmed in his former conclusion. The secondary employed was a rectangle 180 cm. long and 75 cm. wide, and only a length of 75 cm. out of the total length of 510 cm. was changed. The diameter of the wires was 2 mm., and the spark micrometer was used to test for resonance.

In a later paper † on the "Finite Velocity of Electromagnetic Actions," he compares the rate of propagation along copper wires of various diameters, and also the rate of propagation along copper wires with that along iron wires.

He says: "If we replace the copper wire previously used (diam. 1 mm.) by a thicker or a thinner copper wire, or by a wire of another metal, the nodal points are found to remain in the same positions. Thus the rate of propagation in all such wires is the same, and we are justified in speaking of it as a definite velocity. Even iron wires are no exceptions to this general rule; hence the magnetic properties of the iron are not called into play by such rapid disturbances." ‡ (100,000,000 reversals per second.)

---

* Electric Waves, p. 36.

† Poggendorff, Annalen, XXXIV. 551. 1888.

‡ Electric Waves, p. 113.

Professor Oliver J. Lodge remarks as follows in his " Modern Views of Electricity " (page 101, 1889): "I might go on and say that iron makes an enormously worse conductor than copper for rapidly alternating currents. So it does for currents that alternate with moderate rapidity — a few hundred or thousand a second — like those from a dynamo or telephone; but, singularly enough, when the rapidity of oscillation is immensely high, as it is in the Leyden jar discharges and lightning, iron is every bit as good as copper, because the currents keep to the extreme outer layer of the conductor, and so practically do not find out what it is made of."

And again in more detail on page 46 of his " Lightning Conductors and Lightning Guards " (1892) we find the following: " But every one will say — and I should have said before trying — surely iron has more self-induction than copper. A current going through iron has to magnetize it in concentric cylinders, and this takes time. But experiment declares against this view for the case of Leyden jar discharges. Iron is experimentally better than copper. It would seem, then, that the flash is too quick to magnetize the iron; or else the current confines itself so entirely to the outer skin that there is nothing to magnetize."

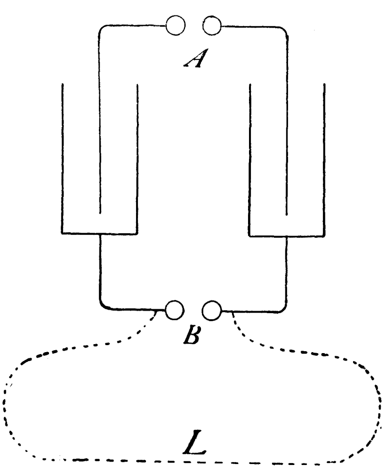

FIG. 1.

The experiment given to substantiate this conclusion is that of the alternate path, as shown in Figure 1. The Leyden jars are charged by an electrical machine, and when a spark occurs at A the charges on the outer coatings may combine by sparking across B or flowing around L. For the path L was used a strip of tinfoil 21 feet long and 3 inches broad, in one case zigzagged backwards and forwards with paraffine paper insulation to abolish self-induction as far as possible, and in the other case wound upon a glass tube to produce as much self-induction as possible.

When the path L was made by the tinfoil zigzag, the critical distance at B, when sparks sometimes passed and sometimes failed for a

given spark distance at A, was 0.6. When the zigzag was replaced by the spiral, the critical spark length at B rose to 6.4. A bundle of finely divided iron was now inserted in the spiral, and the critical length continued still to be 6.4.

He remarks on this result as follows: "Here is magnetic time-lag raised to an extreme. . . . It may be said that the iron fails to get magnetized because of the opposing action of the inverse 'Foucault' current induced in it, just half a period behind the inducing currents. I thought this would be so, of course, with thick iron rods, but with a bundle of thin wires I felt doubtful. . . . Whatever the explanation, the fact of time-lag is patent. Yet there is something strange about it, for that a steel knitting-needle can be magnetized by discharging a Leyden jar round it is mentioned in every text-book, and it is certainly true. There are points here requiring further examination." *

So far the investigations that had in view the effect of iron upon extremely rapid rates of oscillation have given but negative results, though both investigators quoted expected to find that the magnetic properties of the iron would be shown under such conditions. Some positive results showing that the magnetic properties of iron still have some effect upon rapid electric discharges have been obtained by the following observers.

Professor John Trowbridge has proved † that the magnetic character of a conductor is by no means unimportant with 1,000,000 double oscillations per second. In brief the experiment and the results were as follows. The oscillating circuit consisted of a Leyden jar and two parallel wires 30 cm. apart and 510 cm. long. These parallel wires could be replaced by others of different diameter and material. A spark micrometer with tin terminals was included in the circuit, and when the discharge occurred the spark was photographed by means of a rapidly revolving mirror.‡

The following results bear upon the subject under investigation. When the parallel wires were of copper (diam. 0.087 cm.), the number of double oscillations on the negative averaged quite uniformly 9 or 9.5; but when an annealed iron wire (0.087 cm. diam.) was substituted, only the first return oscillation was distinctly visible, with sometimes a trace of the first duplicate.

* Lightning Conductors and Lightning Guards, p. 48.

† Proceedings of the American Academy, XXVI. 115.

‡ Ibid., XXV. 109.

With copper wire (diam. 0.027 cm.) five complete oscillations were quite uniformly visible, but with iron wire (diam. 0.027 cm.) only the first return discharge after the pilot spark was faintly visible.

The time of the double oscillation for the large-sized copper wire was 0.0000020 sec., and for the small copper wire 0.0000021 sec. The author concludes that the magnetic permeability of iron wires exercises an important influence upon the decay of electrical oscillations of high frequency, and that currents of such frequency as occur in Leyden jar discharges magnetize the iron. The data were not sufficient to determine whether there was a change of period, but showed that it must be small if such an effect was produced.

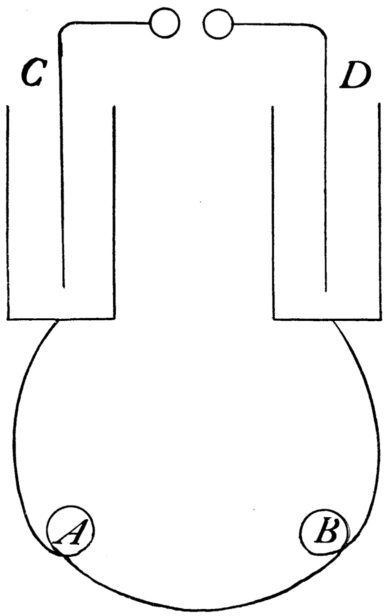

FIG. 2.

Professor J. J. Thompson has stated that the presence of iron can affect the rapidly oscillating electric discharges through a rarefied gas. His method of showing the phenomena is given in Figure 2. C and D are Leyden jars with a spark-gap in the circuit joining their inner coatings, and A and B are two loops in the circuit joining their outer coatings. In the loop A is placed a bulb exhausted to such a degree of sensitiveness that a small change in the electromotive intensity acting upon the bulb produced a considerable effect upon the appearance presented by the discharge. If, when the bulb at A is brilliantly illuminated by the discharge through it, an iron rod be placed in B, the discharge in A ceases; but if a brass rod is placed in B, the discharge in A is unaffected.

The author says: "A striking illustration of the difference between iron and other metals is shown when we take an iron rod and place it in B, the discharge in A immediately stops; if now we slip a brass tube over the iron rod, the discharge in A is at once

restored. If, on the other hand, we use a brass rod and an iron tube, when the rod is put in without the tube the discharge is bright. If we slip the iron tube over the rod, the discharge stops." *

In a paper upon the "Absorption Power of Metals for the Energy of Electric Waves," † Bjerknes has given some results which show the great damping power of magnetic metals upon electric oscillations of very high frequency (100,000,000 double oscillations per second). The apparatus used was, in a slightly modified form, the Hertz vibrator and circular resonator, but in place of the spark micrometer in the resonating circuit he used a much more exact and sensitive arrangement, — a kind of quadrant electrometer with two quadrants to which the ends of the resonating circuit were directly joined. He employed among others resonators of copper, iron, and nickel identical in size and construction. The length of wire in each case was 123 cm. and the diameter 0.5 mm. The length of wire joining the plates of the Hertz vibrator could be varied at pleasure. By varying this, the length of wire necessary for best resonance was found in each case, and the electrometer throws were observed for five different lengths of the primary circuit, including the one for best resonance effects. The graphic representation of these results shows plainly that the metals differ greatly in their power of damping electric oscillation. The electrometer throws were much smaller for the iron and nickel than for the copper, and the curves for iron and nickel come less sharply to a maximum. He further conclusively shows that the damping power of the metals experimented upon increases with their resistance and magnetic susceptibility, and concludes that the magnetic properties of iron and nickel are called into play by their extremely rapid alternations of the magnetizing forces. He notes the fact that the maxima for iron and nickel seem somewhat displaced to one side, which may indicate a greater period, but says that such a displacement of the maxima enters in case of greater damping, so that best resonance does not correspond to exactly equal periods of the two circuits, and adds that a quantitative investigation is necessary to determine to which of the two causes the effect may be referred.

From this brief survey of the field, it is seen how, with more exact and refined means of measurement, some of the results expected, when magnetic metals replaced copper in circuits through which rapid electric discharges were taking place, have finally been observed.

---

* Phil. Mag. (5.), XXXII. 456. July, 1891.

† Poggendorff, Annalen, XLVII. 69. 1892.

Thompson has shown that iron absorbs more energy than brass when subjected to rapidly alternating magnetizing forces, while Trowbridge and Bjerknes agree in showing that iron and nickle conductors damp out electric waves much more rapidly then copper, and all agree that the magnetic properties of iron are called into play under such conditions.

It has seemed to the writer that it was still an interesting and profitable field to investigate whether the magnetic properties of iron were acted upon sufficiently, and in such a way as to produce a change in self-induction that would affect the rate of propagation of electric waves along iron wires. With this end in view, it was sought to simplify some of the apparatus hitherto employed, and to arrange it so that the effect, if observed, could not be due to any other cause than the magnetic properties of the metals experimented upon.

Description of the Apparatus.

For producing the oscillations, the ordinary Hertz vibrator (Fig. 3) was employed, consisting of two plates of zinc, each 40 cm. square, 61 cm. apart, and mounted upon insulated wooden supports. These supports had square wooden bases that could slide in a grooved plank, so that their distance apart could be easily varied, and by turning the supports through a quarter revolution the plates could be made to face each other or to stand in the same vertical plane. The plates were joined by a conductor of brass 0.5 cm. in diameter, which was connected to the plates by sliding with friction into brass tubes 15 cm. long soldered to the plates. The conducting wire was broken by a spark gap, provided with brass balls 3 cm. in diameter. Tin balls were tried, and had perhaps a little greater effect in exciting oscillations in the secondary; but they required polishing fully as often as the brass balls. Finally brass balls with platinum faces were used, and were found to be much more constant in effect than either brass or tin. A circular piece of platinum 0.025 cm. thick and 2 cm. in diameter was wedged into a shape to fit the front of the ball and there countersunk so that the joint was smooth. At first a piece of platinum 1 cm. in diameter was tried, but this was too small, and frequently the sparks jumped from the edge of the platinum or brass. These were discarded, and the larger platinum faces employed. In a very few minutes after polishing they would often reach a state that would remain constant through a long series of observations, but frequently several trials would have to be made before such a satisfactory condition was obtained. All the

observations given below were made with the platinum-faced balls. For polishing the balls a chamois skin with a very small amount of rouge was used, and sometimes they were finally cleansed with alcohol, but this seemed to be no great advantage.

On each side of the spark gap was a hard rubber vertical rod, through which the conducting wire was passed and clamped by a screw. These made a firm support, and held the balls rigorously at the fixed distance apart. The leading wires from the induction coil were soldered to closely fitting short brass tubes that were passed over the conducting wires and rested against the rubber supports. They always remained in the same position, and did not need to be disturbed when the balls were removed for polishing.

A large induction coil (53 cm. long and 19 cm. in diameter) was used to charge the plates of the vibrator. To excite the coil five storage cells were employed, which worked with uniformly good results. The coil was capable of giving a spark 15 cm. long with this source of electromotive force. A sparking distance of 4–6 mm. was found most effective in producing oscillations in the secondary circuit.

The particular feature of the apparatus that applied directly to the investigation was the secondary circuit. In the previous determinations of the wave length due to the Hertzian vibration, the arrangement originated by Hertz and modified by Lecher * and by Sarsin and De la Rive † has been generally employed. In this arrangement secondary disks were placed face to face with the plates of the vibrator, and to each secondary disk a long wire was attached, and these wires then carried through the air parallel to each other, with sometimes an additional disk on the free ends.

With such an arrangement no exact adjustment of the length of the secondary circuit was required in order to excite vigorous oscillation in it, for the direct electrostatic induction between the plates of the primary and the disks on the ends of the secondary wires was so great that powerful oscillations were produced along the secondary wires, whatever their length might be, and several systems of waves could be detected which seemed to give experimental grounds for believing that the wave system sent out from the Hertzian vibrator was very complex.

The capacity of the vibrator is increased by the presence of these

---

* Poggendorff, Annalen, XLI. 850.

† Archives des Sciences Physiques, XXIII. 113. 1890.

secondary disks so near to the plates of the vibrator, so that the wave length found under these conditions is not due to the simple Hertzian vibrator, but to a very heavy complex oscillating system with somewhat obscure internal reactions. Lecher calls attention to the change in the sound of the spark when the two parallel wires of the secondary circuit are bridged across by a conductor, and there is a very marked difference in the spark when the secondary circuit is removed entirely, the spark losing much in body and explosive character. The secondary under these circumstances must exert a strong reaction upon the primary.

It seemed desirable to devise some arrangement depending more directly upon the principle of electrical resonance, and one whose use would not increase the capacity of the vibrator and whose reaction upon the vibrator would be a minimum. This was accomplished by omitting the secondary disks and using simply one wire, as shown in Figure 3.

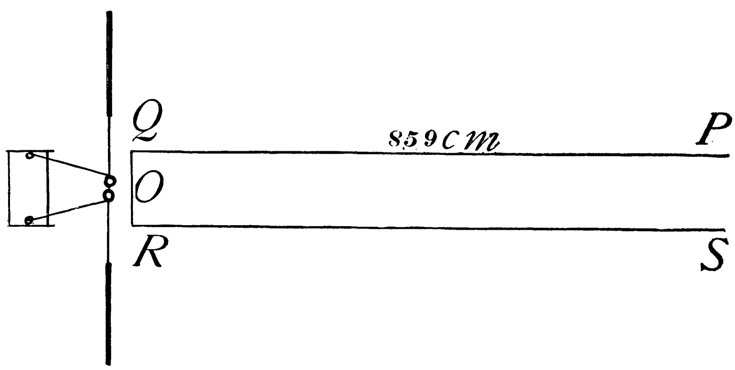

FIG. 3.

The secondary circuit consisted of the long rectangle P Q R S, which was carefully adjusted to resonance by placing the exploring terminals of the bolometer (described later) at P S, and then cutting off the ends of the wire until the length was found that would give the maximum effect. Such a maximum was found when P Q was 859 cm. long. The maximum was sharp and unmistakable, the effect falling off rapidly when the wire was either lengthened or shortened. The result is shown graphically in Figure 4, where, as in all the curves given, distances from Q are used as abscissas and deflections of the galvanometer as ordinates.

To determine the character of the vibration along the wire, the lengths P Q and R S were fixed at 859 cm., the exploring terminals were moved along the rectangle, and the bolometer reading taken for each position of the exploring terminals. The graphic representation of the results is shown in Figure 1 of the Plate. The character of the curve indicates a simple form of vibration. The total length of the wire is equivalent to seven half wave lengths. The minimum points are very nearly the same distance apart, and the distance from the minimum occurring at 748 cm. to the centre of the side Q R may be taken as three half wave lengths. This furnishes a ready means of calculating the half wave length: —

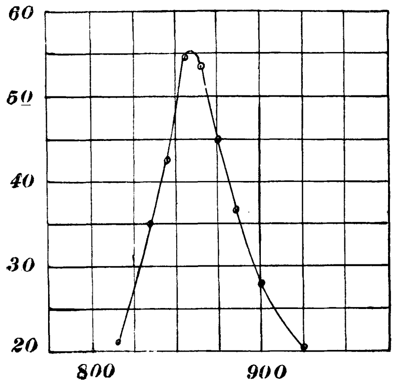

FIG. 4.

$$Q\,R = 30 \text{ cm.} \qquad 748 \times 15 = 763 \text{ cm.}$$

$$763 \div 3 = 254.3 \text{ cm.} = \text{a half wave length.}$$

The distance from this minimum to the end of the wire at P should be one fourth wave length, or 127.15 cm. The actual distance is $859 - 748 = 111$ cm., so that the correction due to the free end of the wire is about 16 cm.

To adjust the length of the wire under this arrangement was a work of considerable difficulty; for in finding the points of maximum effect many trials had to be made, and the wire cut off a few centimeters at a time and then renewed many times. To remove this source of inconvenience, the ends P and S were wound on wooden bobbins, so that shortening and lengthening could be produced without cutting the wire. This was a marked improvement, but the changing size of the coils, as the wire was shortened or lengthened, varied the capacity at the end slightly, and somewhat irregularly. This led to the adoption of the arrangement shown in Figure 5.

The secondary circuit consisted of the rectangle K L M N. The side L N was open, and the lengths of the sides K L and M N could be varied between a few centimeters and ten meters. The ends of the wires K L and M N were in reality formed by the small copper

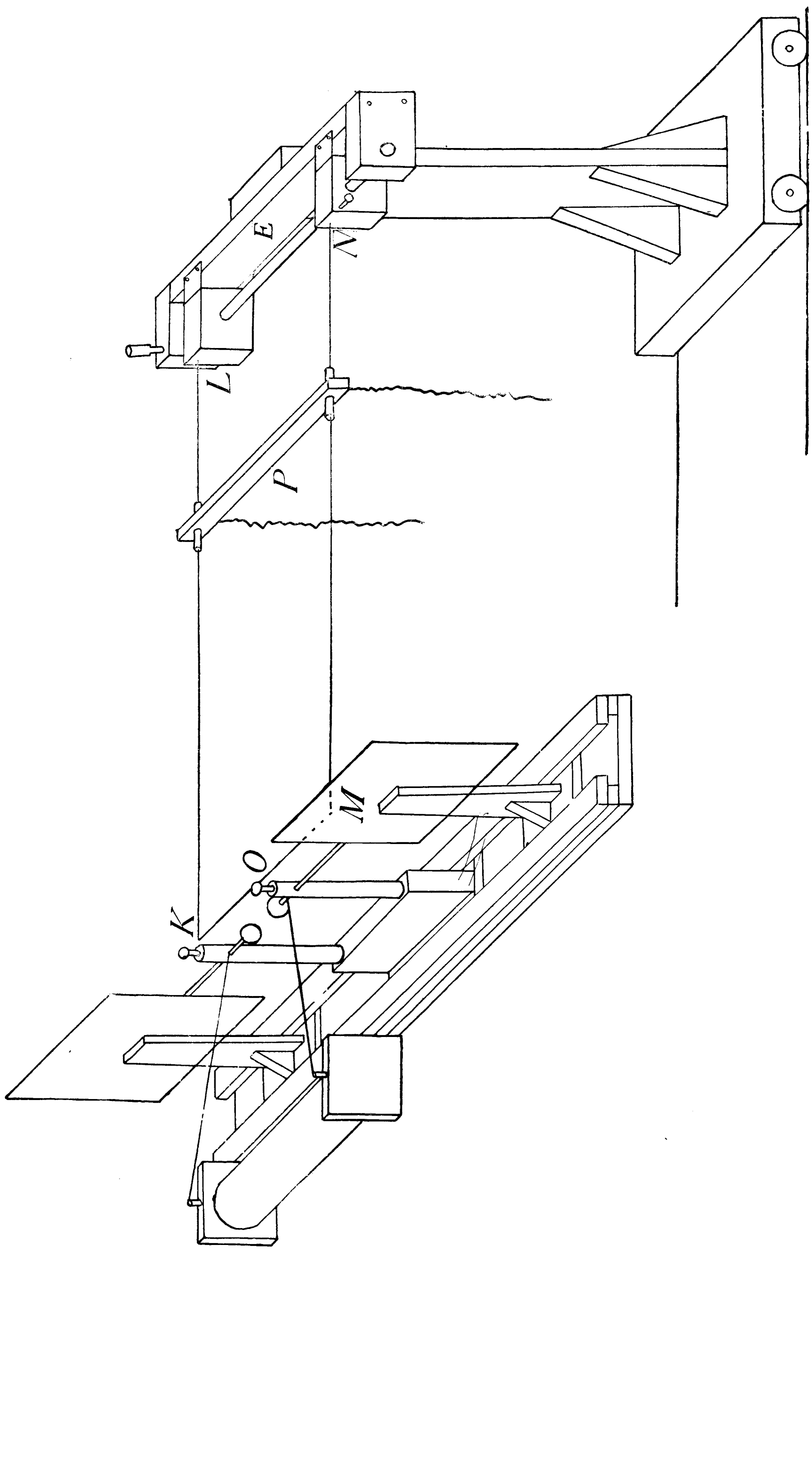

FIG. 5.

boxes L and N. These were 10 cm. square and 4 cm. thick, and mounted upon the wooden bar E by insulating supports. Within the boxes were wooden bobbins fixed to a hard rubber axle, and each capable of holding 10 m. of the largest wire experimented upon. In the front of each box was a small opening for the passage of the wire, but, to assure a firmer contact between the wires and the boxes, a brass block was soldered on the inner side of the front and a binding screw passed in from the outer side of the box. The bar E was fastened to a wooden support resting upon the car, which ran on a wooden track extending the entire length of the room. The car carried a brake, so that the wires could be drawn taut, and the wooden screw held the axle from turning. With this arrangement the length of the wires could be varied at will, while the end capacities would remain constant. The end capacities are not a feature desirable for their own sake, as they destroy the perfect simplicity of the plain rectangle and seem to detract somewhat from the sharpness of the maximum; but the gain in convenience, and the possibility of obtaining a large number of observations whose average values can be used, quite overbalance these considerations in most cases where the apparatus may be applied.

In the early part of the investigation the "Foucault" interruptor was used, but it was extremely irregular in its action, and caused endless annoyance. It ran at an ever varying rate, and required repeated adjustment and constant attention. To remedy at least some of these defects, an interrupter actuated by a small electric motor was devised.* The results obtained from this motor-interruptor were so satisfactory that a detailed description is added.

A Porter's motor, No. 1 (Fig. 6, M), was used to produce the motion. This was actuated by the current from two storage cells, and it ran at a fairly constant speed. The armature of the motor was wound in three sections, and was thus free from dead points, giving it the great advantage for the present purpose that it could be set in motion simply by closing the circuit, making it possible to control it from the observer's station. The motor was geared to the two-crank shaft K by means of a wheel and pinion. The wheel and pinion had the ratio of 144 to 24 in the following investigation, but the motor could slide on the brass bed-plate so that pinions of other sizes were available. The speed of the shaft K was about 750 revolutions per min-

* This and the other apparatus especially prepared for this investigation, and requiring much technical skill in its construction, was made by the mechanician of the laboratory.

ute, and about 25 breaks were produced per second. The plunging rods were thinned down at O so that they were flexible and gave the required freedom of motion; they ran through the bed-plate and the brass bar below, which served as guides. The plunging rods carried

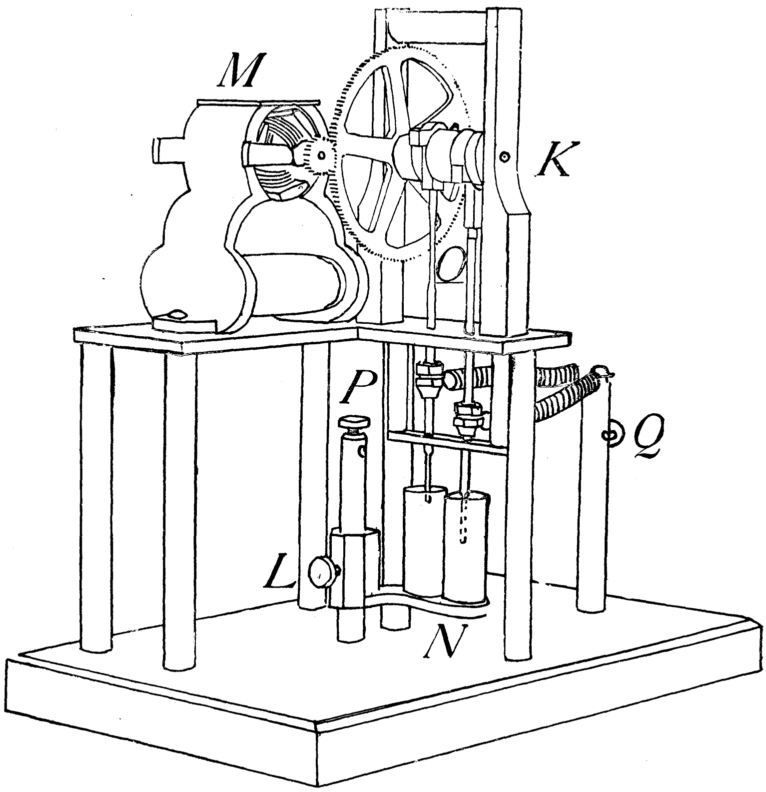

FIG. 6.

lock-nuts by which the flexible coils leading the current from the brass post Q were attached. The lower ends of the plungers were of platinum wire, No. 18. The glass mercury cups had brass bottoms that screwed into the brass arm N, which was adjustable by means of the collar and binding screw L along the pillar P. At P was attached one pole of the battery actuating the coil, and also one pole of the condenser in the base of the coil, and at Q was attached one pole of the coil and the other pole of the condenser.

The cups were filled with mercury to a height of about 8 mm., and then filled with alcohol to within a few millimeters of the top. They usually required cleaning only after several hours' use, when the surface of the mercury consisted of very fine globules, and sharp breaks of the circuit were not made at each stroke of the plunger, as was indicated by the occasional failure of the spark. The length of the spark was kept constant during the observation, and the character of the spark depended much upon the exact adjustment of the height of the mercury cups of the interruptor. While it and the coil were both in action, the height of the cups was adjusted until the sparks came regularly with a peculiar crashing snap, and showed the bluish white thick body that Professor Hertz described. Both the ear and eye were soon so trained that they gave quick and sure information of the character of the spark, but the ear was better than the eye. The exact height of the mercury cups was of the utmost importance, a slight difference in height changing the character of the spark greatly. The exact point was best found by concentrating the attention upon the sound, and then slowly raising or lowering the cups, when suddenly there would seem to be a rhythm between the sound of the interruptor and the snap of the spark, which would disappear with further motion up or down. This rhythm would indicate that for every break of contact there was a corresponding pilot spark between the micrometer balls.

For measuring the effects produced in the secondary circuit, the bolometer as designed by Paalzon and Rubens * was used with most satisfactory results. The bolometer was constructed according to the description given by them in the paper referred to, and differed only in minor details arising from the circumstances and the materials obtainable. The accompanying diagram is theirs, but the following description applies in all its details only to the instrument constructed for this investigation.

The bolometer is in reality a double Wheatstone bridge. The four arms of the bridge are the resistances W_1, W_2, W_3, W_4, of which W_1 and W_2 are quadrilateral circuits of equal resistance, and W_3 and W_4 are coils of equal resistance. The quadrilateral A B C D, or W_2, is really a small Wheatstone arrangement, used as one branch of the main bridge. For convenience in description, W_2 will be called the "bolometer branch," and the term bridge limited to the Wheatstone

* Anwendung des bolometrischen Princips auf electrische Messungen. Poggendorff, Annalen, XXXVII. 529.

net, of which W_2 is a portion. If in the quadrilateral A B C D the sides are of equal resistance, and if then a current be led in at B and out at D, or *vice versa*, no difference of potential will be produced between A and C; in the same way, a current can be led in at A and out at C without producing any potential difference between B and D. The two currents can traverse the rectangle simultaneously and exert no effect upon each other arising from difference of potential. If, after the bolometer branch and the bridge are both balanced, a current, alternating or direct, is sent through the bolometer branch W_2 from B to D, none of the current will pass through the galvanometer; but the resistance of the branch W_2 will be increased by the evolution of heat, and the bridge thrown out of balance, as indicated by the deflection of the galvanometer.

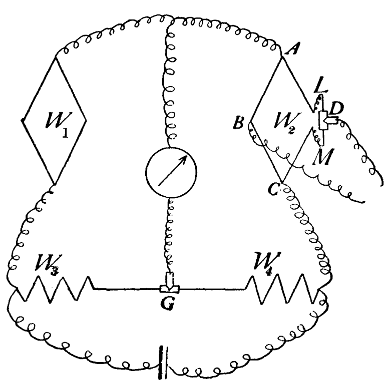

FIG. 7.

The resistances W_3 and W_4 were coils of fine German silver wire double wound on wooden spools, and of three ohms' resistance each. The quadrilaterals W_1 and W_2 were of iron wire, radius 0.035 mm., and each side had a resistance of three ohms. The arms of the bolometer branch W_2 were balanced before use by means of the sliding mercury contact D, so that no deflection was produced in the galvanometer when a steady current was passed from B to D. The sliding mercury contact consisted of 20 cm. of No. 18 German silver wire, L M, and a sliding block of brass, D, which contained in its upper surface a small cup-shaped cavity filled rounding full with mercury. The German silver wire was amalgamated to insure a good contact. The contact G was of similar construction. The connections were of No. 18 copper wire, whose resistance was negligible in comparison with the bridge arms.

The adjustment of the bolometer branch, once made, remained constant through the series of observations, but the bridge adjustment by means of the sliding contact G had to be made frequently. To supply the bridge current a Daniell cell was used with a resistance of 5 to 30 ohms in circuit.

The galvanometer was of the Thompson type and had a resistance of 56.9 ohms, and a figure of merit of 2.3×410^{-8}, with a scale distance of 1 m. and scale divisions of 1 mm. The time of a half vibration of the needle was 7 seconds.

The bolometer resistances were attached to the under side of the cover of a well made box of cherry (33 cm. long, 26 cm. wide, and 8 cm. deep). The resistances W_1 and W_2 were supported on slender posts, so that they came nearly in the middle of the box, and were equally exposed to the air on all sides. On the upper side of the cover were the binding posts and sliding contacts. This box was placed inside another one of similar construction (47 cm. long, 39 cm. wide, and 23 cm. deep), and the space between was packed with cotton wool. Cords from the sliding contacts were led through the sides of the outer box, so that adjustments of the arms could be made without exposing or touching the bolometer. Even with this protection from thermal changes, much difficulty was experienced when the hot weather came, and the temperature of the room could not be kept constant. It was found, however, that early in the morning very little disturbance was experienced, and most of the observations whose results are reported were recorded before the heat of the morning was much felt in the room containing the bolometer.

To use the bolometer for measuring the energy at all places on the wires forming the secondary circuit, the arrangement devised by Rubens * was used. Over the two wires K L and M N (Fig. 5), were slipped thick-walled capillary tubes of glass, about 5 cm. long, and these were held by a small wooden bar, so that they could be slipped along together. This arrangement is spoken of hereafter as the exploring terminals. The ends of the lead-wires to the bolometer were wound once around the tubes and fastened by sealing wax. These formed two Leyden jars of extremely small capacity, and the oscillations of charge on their inner coatings, which were formed by the wires K L and M N, produced rapidly oscillating currents through the bolometer resistance W_2. For leading wires to the bolometer silk covered copper wire No. 36 was used that had been drawn through hot paraffine. Larger wires were tried and wires with rubber insulation, but with these the apparatus was less sensitive.

The following method was pursued in taking the observations. The interruptor was set in action, the circuit closed through the induction coil, and an observation taken of the first swing of the

* Ueber stehende electrische Wellen in Drähten und deren Messungen. Poggendorff, Annalen, XLII. 154.

needle. The circuits were broken as soon as the needle reached the end of its first swing, and the extent of this excursion was the reading recorded. In accordance with the experience of Paalzow and Rubens, it was found that a steady deflection could not be obtained, but this first swing was, under like conditions, quite constant, and a preliminary calibration of the instrument by passing currents of known strength through the bolometer branch W_2 showed that the square root of the deflection was in a constant ratio to the current. The needle was quickly damped by making and breaking the circuit through the induction coil, with the interruptor in action.

The rooms at disposal were very suitable for such an investigation. The main room was 18 m. long, 6 m. wide, and 5.5 m. high, and it contained a very small amount of metal, and as it was in the non-magnetic part of the laboratory that small amount was of brass and bronze except the temporary addition of a small steam radiator in the corner back of the oscillator. The oscillator was placed at 4.5 m. from one end, and the parallel wires ran through the middle of the room at a distance of 1.6 m. from the floor. The leading wires carrying the currents from the batteries were of twisted cable and placed high up against the walls. The bolometer and galvanometer were in an adjoining room, where the observation table was equipped with the keys necessary for complete control of the interruptor and induction coil. By this means it was possible for one person to carry on the investigation, though it was very trying. Not only were the observer's eyes in use, but it was necessary to listen intently to the sparking of the oscillator, as after some experience very slight changes could be detected and a close judgment formed of the steadiness of the spark; besides, it was necessary to note the sound made by the interruptor, as small variations in its speed were easily noticed. The only time the interruptor was likely to show much change in speed was when the battery was beginning to fail, or the brushes had become worn.

The theory of the investigation was based upon the principle of electrical resonance.

Bjerknes has shown, in the paper previously noted, that, if damped electromotive impulses obeying a sine law be assumed to act upon a secondary circuit, there will be produced in the secondary circuit oscillations of the period belonging to the primary impulses, and at the same time oscillations proper to the secondary circuit, and that these induced oscillations will reach their maximum amplitude when the two circuits have the same period. His investigations also show that the oscillations of the Hertz vibrator damp out much more rapidly than

the oscillations in the secondary, at least when there is no spark gap in the secondary circuit.

The sides of the rectangle (Figure 5) were reduced to a few centimeters in length, so that it could be safely assumed that its period was much shorter than that of the vibrator. The plates of the vibrator were fixed at 61 cm. apart, and the side K M of the rectangular secondary was placed at 6 cm. from the conductor joining the plates of the vibrator, with its centre O opposite the spark gap. The sides K L and M N lay in the horizontal plane through the axis of the vibrator, and were held by the end supports at 30 cm. apart. The apparatus was symmetrical about a vertical plane through its spark gap normal to the axis. The exploring terminals were kept at L N and bolometer readings taken for each small addition to the length of the sides K L and M N. When best resonance was obtained with the shortest length of the secondary circuit that gave a maximum, it was assumed that the secondary had the same period as the primary, and that its equivalent length was a half wave length, its actual length depending upon the effect due to the free ends. The occurrence of resonance is a very marked phenomenon, even with a vibrator that damps as rapidly as the Hertzian. The following table shows two series of readings for the first maximum when an iron wire was used: —

Length of side of rectangle	15	25	35	40.0	42.5	45.0	50	60	75
Deflections	107	145	156	194.3	199.2	181.5	140	81	42
Deflections	94	119	161	185.0	191.0	178.0	136	76	34

There can be no free motion of the electricity at the ends of the secondary circuit, but an accumulation alternately positive and negative and the resulting alternation of potential, the phase at L being always opposite to the phase at N. Elsewhere along the circuit the electricity moves with more freedom and less accumulation. The point O may be called the electrical middle of the circuit, where the accumulation is least and the movement most restrained. The electromotive impulses from the vibrator act directly upon the side K M, so that O remains a point of free motion, or the central segment of the wave, while L and N are always places of no electric motion, or the nodal points. The nodes under this view are the places of greatest potential difference, so that in the graphic representation of the results the maximum points of the curves correspond to the nodes, the bolometer throws being the largest when the exploring terminals are placed where the potential difference is greatest. The shortest circuit being

a half wave length long, a second resonating circuit ought to be found by increasing each side of the rectangle by a half wave length, making the circuit 3 half wave lengths long, and a third when the circuit is 5 half wave lengths long, and so on.* This is evident from a consideration of the accompanying diagram. O is the center of a central segment and the points marked 1, 3, 5, are always nodes.

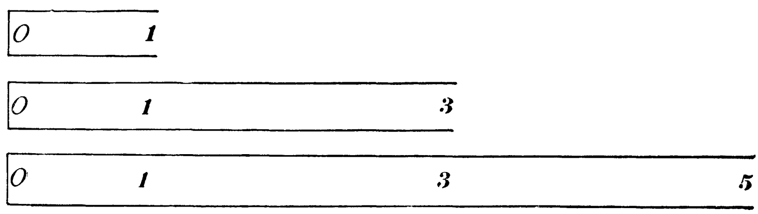

FIG. 8.

From the results that he obtained, Bjerknes concluded that the change of period, if there was such a change, by the use of iron in place of copper, could not exceed two per cent.

The difference in length between a copper and an iron circuit of the same period would be very small with circuits a half wave length long, but this difference would be three times as great with circuits 3 half wave lengths long, and there might be a cumulative difference that would finally become measurable by the use of circuits of still greater length. This theory was tested in the following way. A copper wire (diameter 0.1201 cm.) was used as the secondary circuit in Figure 5. The sides were taken 15 cm. long, and then gradually lengthened to 875 cm., and bolometer readings observed for each addition. The results are shown graphically by the upper curve in Figure 4 of the Plate. The critical points in the curve are the results of many separate determinations. The unsteadiness of the spark in the vibrator made the determinations somewhat laborious, though a single series of observations would locate a maximum very closely. After this had been done, a space of about a meter including the maximum point was worked over forward and back, changing its length 2.5 to 5 centimeters at a time in the region of the maximum. To assure the steadiness of the spark during such a series of observations, some convenient length of circuit was chosen as a point of reference, and observations taken before and after the series; if these showed that the activity of the spark was practically the same, the readings of the

* J. J. Thompson, Recent Researches in Electricity and Magnetism, § 297.

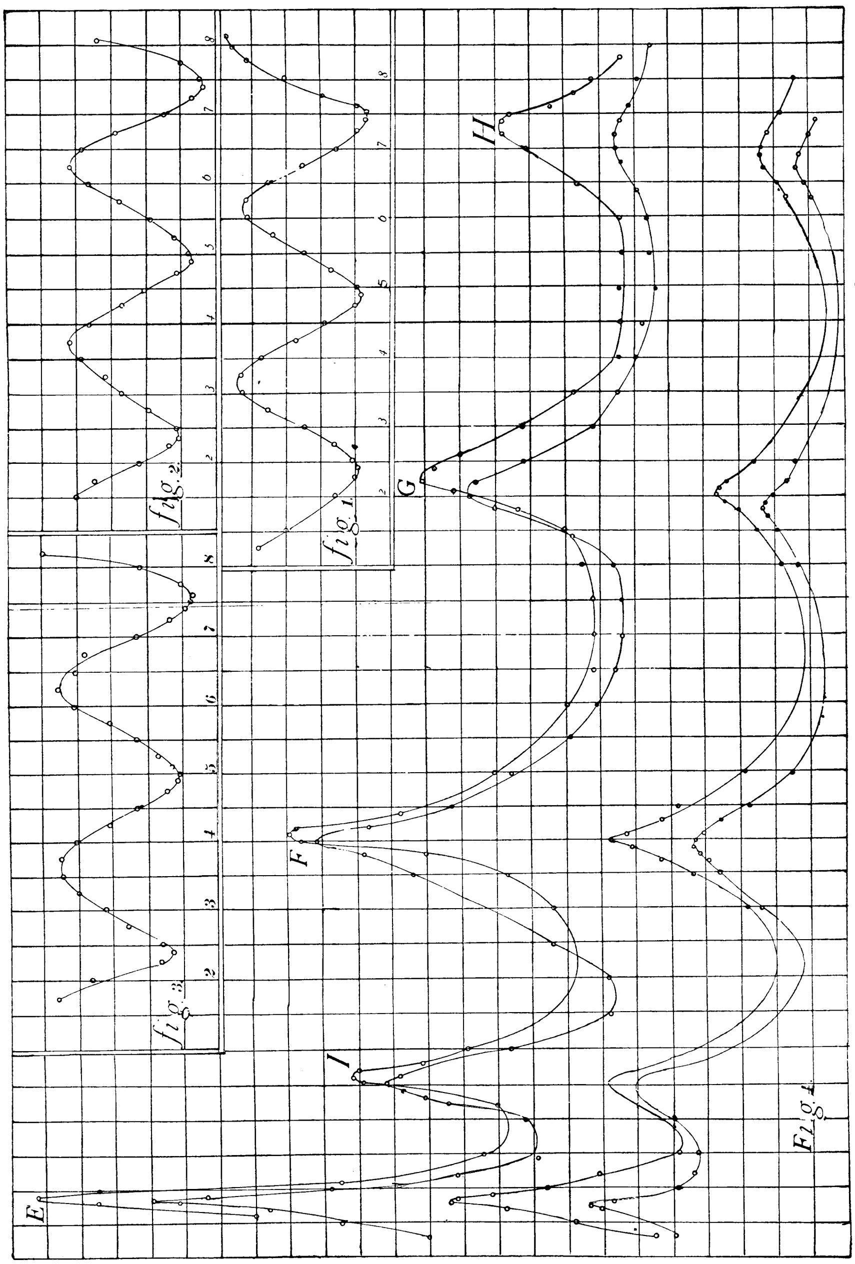
fig. 1
fig. 2
fig. 3
Fig. 4
E
F
G
H
I
1
2
3
4
5
6
7
8

series were retained. The results here given rest upon such readings.

An examination of the curve shows four maxima, E, F, G, H, occurring when the sides of the rectangle were 45, 306, 562.5, and 818 cm. long. The additions of wire for the successive maxima after the first were 261, 256.5, and 255.5 cm. These additions should be a half wave length; the last two are nearly the same, but the first differs by 5 cm. from the average of the last two, which is 256 cm. With the sides fixed 818 cm. long, the wave form along the circuit was determined by sliding the exploring terminals over the wires by short steps, and observing the bolometer throws for each position. The result is shown in Figure 3 of the Plate. The critical points were determined several times, and a method similar to that described above was used to assure the constant activity of the spark. The curve shows three minima, occurring at 240, 496, and 752 cm. Starting from the point O these give half wave lengths of 255, 256, and 256 cm., with an average of 255.6 cm. The third minimum at 752 was determined with care, as it was to be used as a basis for calculating the half wave length. A small error in determining the position of this miniumm would be divided by 3 in obtaining the result, since its distance from O was 3 half wave lengths. The total length of the circuit was 7 half wave lengths. From the third minimum to the end it was one fourth of a wave length, the capacities bring each equivalent to 62 cm. of the wire. By fixing the length of the rectangle at 562.5 and 306, a similar investigation showed the circuits to be respectively 5 and 3 half wave lengths long.

An explanation of the fact mentioned above, that the distance between the first and second maxima was anomalously large, may possibly be this: the sides of the rectangle for the first maximum were but 45 cm. long, so that the effect of the closed end in increasing the self-induction was relatively large, and the maximum appeared earlier than it otherwise would; but when the rectangle was 300 cm. long, the influence of the closed end became relatively small, and the second and future maxima came in the normal positions. In the first case the capacity was mostly local, while in the second it was largely distributed, and the length of the circuit was greater than the wave length. This same effect appeared in every case, and seemed to be a constant phenomenon.

The maximum I, omitted from the above discussion, was not constantly present, but appeared when the primary spark was particularly active, and seemed to belong to a circuit whose period was to the

period of the primary as 5 to 3. The side of the rectangle was 127.5 cm., and the end capacities equivalent to 62 cm. of wire. The half wave length was

$$30 + 127.5 \times 2 + 62 \times 2 = 409. \quad 409 \div 255.6 = 1.6, \text{ nearly.}$$

This was the only indication observed of complexity in the vibration of the oscillator. It appears that, when the oscillator is especially active, it can excite a circuit having this ratio to itself, or that the vibration is not a simple one. Time was not at disposal sufficient to decide this point, which is left for future investigation.

A comparison of the curve (Fig. 1 of Plate) obtained from the plain wire circuit with the curve (Fig. 3 of Plate) obtained when capacities were fixed on the free ends shows a quite satisfactory agreement in the results, which tends to create confidence in both methods. The half wave length by the first is 254.3 cm., by the second it is 255.6 cm., values which differ by about one half of one per cent. There is a marked difference, as was to be expected, in the form of the curve for the quarter wave length next the free ends. When end capacities were used, the accumulation of charge seemed mainly confined to those out of reach of the exploring terminals, while with the plain wire it seemed distributed over a greater distance, and could be detected by the exploring terminals. In each case the effect of the ends was to make the curve depart from its normal form along the free wire.

An annealed iron wire (diameter 0.1186 cm.) was put in place of the copper, and the same series of observations was made as with the copper. The results are shown graphically in the lower curve of the upper pair in Figure 4 of the Plate. The maxima E, F, G, H, appear at 42.5, 301, 553, and 805 cm.; in each case before the corresponding maxima with the copper, and the difference increases with the length of the circuits, as is evident from an examination of the curves. The successive additions after the first maxima are 258.5, 252, and 252 cm.; the last two agreeing, while the first, as with the copper, is larger. With the sides of the rectangle fixed at 805 cm., the form of the wave was found as shown in Figure 2 of the Plate. The third minimum occurs at 740 cm. Calculating its half wave length as before, $740 + 15 = 755$. $755 \div 3 = 251.6$ cm. This agrees well with the value 252 cm. given above by the last two additions, but differs by 4 cm. from the value found when the copper was used.

The same series of observations was repeated with a second pair of finer wires (diameter of copper wire 0.07836 cm., diameter of iron

0.07850 cm.). The results are shown in the lower pair of curves in Figure 4 of the Plate, the upper one, as before, being the copper. A comparison of the curves shows the same general result, which appears more distinctly from the following table.

	1st Maximum.			2d Maximum.			3d Maximum.			4th Maximum.		
	Cu	Fe	Difference.	Cu	Fe	Difference	Cu	Fe	Difference.	Cu	Fe	Difference.
Upper pair	45	42.5	2.5	306	301	5	562.5	553	9.5	818	805	13
Lower pair	40	57.5	2.5	300	294	6	552	540	12.0	799	784	15

The successive differences should be in the ratio of 1, 3, 5, 7, if the theory of the investigation is correct. The differences for the first two maxima are very small, so that the experimental error in their determination would be relatively large.

In the case of the fourth maximum, the damping was so great that it was difficult to fix the point with certainty. The difference for the third maximum was relatively large, and the determination of the maximum point was sharp. Taking this difference as a point of reference, the calculated and observed values are shown in the following table.

	1st Maximum.		2d Maximum.		3d Maximum.		4th Maximum.	
	Calculated.	Observed.	Calculated.	Observed	Calculated.	Observed	Calculated.	Observed
Upper pair	1.9	2.5	5.7	5	9.5	9.5	13.3	13
Lower pair	2.4	2.5	7.2	6	12.0	12.0	16.8	15

The observed half wave lengths for the four wires were as follows: —

$\begin{cases} \text{Copper (diameter 0.1201 cm.), 255.6 cm.} \\ \text{Iron (diameter 0.1126 cm.), 251.6 cm.} \end{cases}$

$\begin{cases} \text{Copper (diameter 0.07836 cm.), 251.6 cm.} \\ \text{Iron (diameter 0.07850 cm.), 246.8 cm.} \end{cases}$

The wires in each pair were as near the same diameter as could be found, the iron of the larger pair having slightly the smaller diameter,

but the copper being the smaller one in the second case. In other respects the circuits compared were as nearly identical as possible. The capacity per unit length being the same for wires of the same diameter, the shortening of the wave length when iron displaced copper of the same diameter must be caused by an increase in self-induction due to the magnetic properties of the iron. If this is true, it means that the magnetization of iron can be produced and reversed 115 million times per second. This reduces the "time-lag" of magnetization to a very small quantity, if magnetizing forces of such duration are capable of bringing the magnetic properties of the iron into play.

In the case of extremely rapid oscillations, Prof. J. J. Thompson has shown (Recent Researches in Electricity and Magnetism, § 295) that approximately $y^2 = \frac{2}{L' C}$, where $\frac{y^2}{4\pi^2}$ is the square of the frequency, and L' is the self-induction for any rapid oscillations, and C the capacity of the system. It is easy from this to calculate an approximate value for the ratio between the self-induction per unit length of the iron and the copper.

Let L = the self-induction of the copper per unit length.
Let L' = the self-induction of the iron per unit length.
Let C = the capacity of either per unit length.

Using as a basis of calculation the data from the third maximum G of the curves of Figure 4 of the Plate, the total length of the copper circuit (diameter 0.1201 cm.) is: —

The sides,	$562.5 \times 2 =$	1125 cm.
The closed end,	$=$	30 cm.
The equivalent of the end capacities,	$62 \times 2 =$	124 cm.
		1279 cm.

For the iron (diameter 0.1186 cm.) the length is: —

The sides,	$553 \times 2 =$	1106 cm.
The closed end,	$=$	30 cm.
The equivalent of the end capacities,	$61 \times 2 =$	122 cm.
		1258 cm.

Since the two circuits have the same frequency, the products of the self-induction by the capacity are equal.

$$1258^2 L' C = 1279^2 L C.$$

$$\frac{L'}{L} = 1.034.$$

In the same way, for

$$\left\{\begin{array}{l}\text{Copper (diameter 0.0884 cm.)}\\ \text{Iron (diameter 0.08847 cm.)}\end{array}\right. \qquad \frac{L'}{L} = 1.041.$$

$$\left\{\begin{array}{l}\text{Copper (diameter 0.07836 cm.)}\\ \text{Iron (diameter 0.07850 cm.)}\end{array}\right. \qquad \frac{L'}{L} = 1.043.$$

By the use of Lord Rayleigh's formula for induction under very rapid oscillations, it is easy to calculate the permeability of the iron, since the ratio between the self-induction of the iron and copper are given by the previous calculation.

Lord Rayleigh's formula is

$$L' = l\left(A + \sqrt{\frac{\mu R}{2 p l}}\right),$$

where l is the total length of the circuit; A, a constant depending only on the form of the circuit, or $l\,A$ is the self-induction of a similar copper circuit; μ, the permeability; R, the resistance; $p = 2\pi n$, where n is the number of complete oscillations per second.

The value of $p = 2\,\pi\,n = 360{,}000{,}000$.

R for iron wire diameter	0.1186 cm.	= .1328	ohms per sec.
" " "	0.08847 cm.	= .227	" "
" " "	0.0785 cm.	= .301	" "

For iron diameter 0.1186 cm.

$$L' = 1.034\,L = l\left(A + \sqrt{\frac{\mu R}{2 p l}}\right),$$

$$L + .034\,L = L + l\sqrt{\frac{\mu R}{2 p l}},$$

$$.034\,L = l\sqrt{\frac{\mu R}{2 p l}}.$$

Calculating the value of L for a copper circuit l units long, substituting the value in the above equation, and solving, we find: —

For the iron wire diameter	0.1186 cm.	$\mu = 430$
" " "	0.08847 cm.	$\mu = 389$
" " "	0.0785 cm.	$\mu = 336$

These values for the permeability all fall within a reasonable limit, and have for an average $\mu = 385$. These are the values found for different specimens of wire made by the same company, but the specimens were wound and unwound and stretched many times during the series of observations.

Besides the shortening of the wave length due to the increased self-induction of the iron, there is shown a decided increase in the damping, as has already been observed by Trowbridge and Bjerknes. In Figure 4 of the Plate the curves for iron fall below the corresponding ones for copper, but owing to the change in the activity of the primary spark no exact measurement was made. It was only observed that the bolometer throws with the copper circuit were always greater than those with the iron circuit of same dimensions when the spark was constant as far as the eye and ear could judge.

A value for the damping factor $\epsilon^{-\frac{Rt}{2L}}$ can be calculated for the iron and the copper. Lord Rayleigh's formula for the resistance under very rapid oscillations is:

$$R' = \sqrt{\tfrac{1}{2} p \, l \, \mu \, R}.$$

For iron wire circuit (diameter 0.1186 cm.),

$$l = 1258 \,; \; \mu = 430 \,; \; R = 1.67 \times 10^9 \,; \; p = 36 \times 10^7 ;$$

whence

$$R' = 403 \times 10^9 \,; \; L' = 34 \times 10^3 .$$

The damping factor becomes $\epsilon^{-6 \times 10^6 t}$.

The time required for the amplitude to fall to one half of its maximum value is found from the equation $\frac{1}{2} = \epsilon^{-6 \times 10^6 t}$:

$$t = .000000115 \text{ sec.}$$

On the basis of 115,000,000 alternations per second, the number of complete oscillations in this time is 6.5. A similar calculation for the corresponding copper circuit gives nearly sixty times as many.

It has been suggested that the greater damping of the iron might give an apparent change of wave length. If the iron circuit is chosen too short, and the maximum point is sought by adding to the length of the wire, the increase of length would increase the damping and tend to diminish the bolometer throws while the approach to the point of resonance would tend to increase the effect on the bolometer; the two in fact would work against each other. If, on the other hand, the circuit be chosen too long, and resonance is sought by shortening the wire, the two would work together.

If the damping plays an important part we might expect different results under these conditions. Iron wire (diam. 0.08847 cm.) was used, and the circuit shortened until the sides were 15 cm. long and the first

maximum point was found by gradually lengthening the wire; it was then found by gradually shortening the wire from an initial length of 60 cm. for the sides of the rectangle. The results are shown in Figure 9 where the upper curve is based on the data found when shortening the wire, and the lower on the data found when lengthening it. The two differ by less than a centimeter, which is as near as two determinations could be expected to agree. In all determinations of the critical points of the curves shown, readings were taken both forward and back, and the averages used as data for the curves.

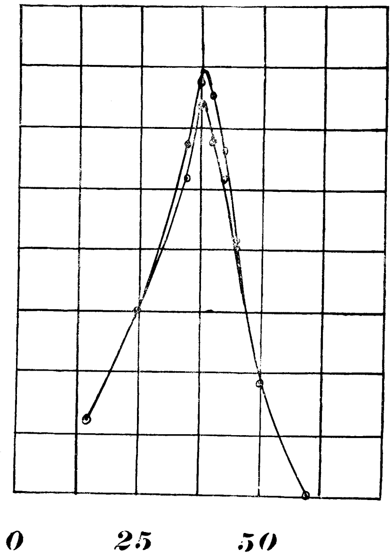

FIG. 9.

Another result of the investigation is apparent when copper circuits are compared in which wires of different diameters are used.

		3d Max G
Copper wire, diameter	0.12010 cm.	562.5
" " "	0.08840 cm.	553.5
" " "	0.07836 cm.	552.0
" " "	0.03915 cm.	535.0

The half wave lengths calculated from this maximum are: —

Copper (.01201 cm.)	255.8 cm.
" (.08840 cm.)	252.2 cm.
" (.07836 cm.)	251.6 cm.
" (.03915 cm.)	244.8 cm.

These are found by taking the total length of the circuit, and dividing by 5.

$535 \times 2 = 1070$ cm., length of wires.
30 cm., length of closed end.
$62 \times 2 = 124$ cm., equivalent of end capacities.
1224 cm.
$1224 \div 5 = 244.8$ cm.

The results here presented differ from those hitherto given, and particularly from those of the late Professor Hertz; but his investiga-

tions were made with the spark micrometer as a measuring instrument, and the same is true of Dr. Lodge's work with the alternate path. The adaptation of the bolometer principle to this purpose furnishes a much more accurate means of determining wave lengths and the occurrence of resonance. It is not surprising that a change of wave length of less than two per cent escaped detection by the spark micrometer method. The difference between copper and iron increases as the diameter of the wires diminishes; with wires 2 mm. in diameter, the size mostly used by Hertz, the difference would be exceedingly small.

The range of wires suitable for the study of the phenomena is rather limited. If the wires are larger than 1 mm. in diameter, the difference between iron and copper is slight; while with wires less than 0.5 mm. in diameter the damping is so great that long wires cannot be used and advantage cannot be taken of the cumulative effect which is the basis of the present method. There is no disagreement between the results here given and those reported by Trowbridge and Bjerknes. The circuits Trowbridge used were so long that the iron damped the oscillation too rapidly, and the circuits used by Bjerknes were so short that the difference between the copper and iron could not be determined with certainty.

I wish to express my great obligation to Professor John Trowbridge for the encouragement and suggestions that I have received from him, and for his kindness in placing the resources of the Jefferson Physical Laboratory so completely at my disposal.

Conclusions.

1. The self-induction of iron circuits is greater than that of similar copper circuits under very rapid electric oscillations (115,000,000 reversals per second). This change in self-induction varies from 3.4 to 4.3 per cent in the present investigation, and increases with decreasing diameters.

2. The increase in self-induction produces greater damping, and a shortening of the wave length of between 1.5 and 2 per cent.

3. The permeability μ of annealed iron wires under this rate of alternation is about 385.

4. For oscillations of the same period, the wave length along parallel copper wires varies directly with the diameter of the wires. Range of wires used 0.03915 cm. to 0.1201 cm. The maximum decrease observed is 5 per cent.

Jefferson Physical Laboratory,
July 24, 1894.